1. **Draw the mode shapes for two rotor system.**

Ans.

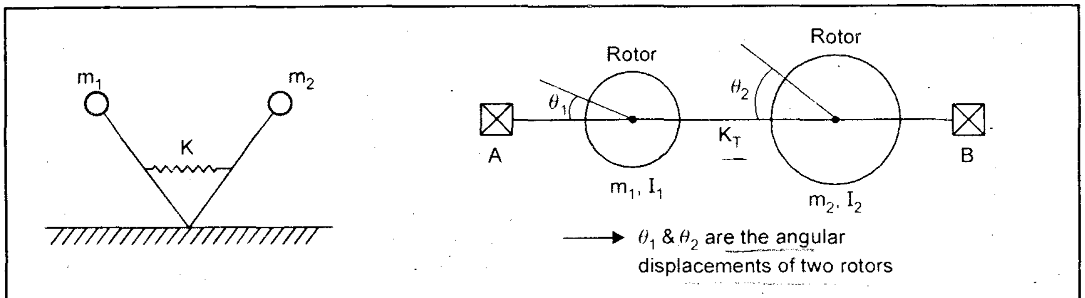


Fig. 5.1 Fig. 5.2

Torsional Vibrations : Consider fig. 5.2. A shaft AB is carrying two rotors of moment of inertia I and ‘2 Let and °2 be the angular displacements of rotor at any .instant from mean position. The equation of motion can be written s, any .instant from mean position. The equation of motion can be written s,

$$I_1 \ddot{\theta}_1 = -K_T(\theta_1 - \theta_2) \quad \text{...(I)}$$

$$I_1 \ddot{\theta}_1 + K_T(\theta_1 - \theta_2) = 0 \quad \text{...(II)} \quad \left[\ddot{\theta}_1 = \frac{d^2\theta_1}{dt^2} \right]$$

$$I_2 \ddot{\theta}_2 = -K_T(\theta_2 - \theta_1) \quad \text{...(III)}$$

$$I_2 \ddot{\theta}_2 + K_T(\theta_2 - \theta_1) = 0 \quad \text{...(IV)} \quad \left[\ddot{\theta}_2 = \frac{d^2\theta_2}{dt^2} \right]$$

Put,

$$\theta_1 = a_1 \sin \omega t, \theta_2 = a_2 \sin \omega t$$

$$\ddot{\theta}_1 = -a_1 \omega^2 \sin \omega t$$

$$\ddot{\theta}_1 = -\omega^2 \theta_1, \text{ similarly, } \ddot{\theta}_2 = -\omega^2 \theta_2$$

Putting these values in (II) and (IV)

$$I_1 \times -\omega^2 \theta_1 + K_T (a_1 - a_2) \sin \omega t = 0$$
$$-I_1 \omega^2 a_1 \sin \omega t + K_T (a_1 - a_2) \sin \omega t = 0$$
$$-\omega^2 I_1 a_1 + K_T (a_1 - a_2) = 0 \quad \text{...(V)}$$
$$I_2 \times -\omega^2 \theta_2 + K_T (a_2 - a_1) \sin \omega t = 0$$
$$I_2 \times -\omega^2 \times a_2 \sin \omega t + K_T (a_2 - a_1) \sin \omega t = 0$$
$$I_2 \omega^2 a_2 + K_T (a_2 - a_1) = 0 \quad \text{...(VI)}$$
$$\omega^2 I_1 a_1 + K_T (a_1 - a_2) = 0$$
$$\omega^2 I_2 a_2 + K_T (a_2 - a_1) = 0$$
$$(K_T - I_1\omega^2)\, a_1 - K_T a_2 = 0$$
$$-K_T a_1 + (K_T - I_2\omega^2)\, a_2 = 0$$

Solving by determinant

$$\begin{vmatrix} K_T - I_1\omega^2 & -K_T \\ -K_T & K_T - I_2\omega^2 \end{vmatrix} = 0$$

$$(K_T - I_1 \omega^2)(K_T - I_2 \omega^2) - K_T^2 = 0$$
$$K_T^2 - K_T I_2 \omega^2 - K_T I_1 \omega^2 + I_1 I_2 \omega^4 - K_T^2 = 0$$
$$\omega^2 (I_1 I_2 \omega^2 - K_T I_1 - K_T I_2) = 0$$
$$\omega^2 = 0$$
$$\boxed{\omega_1 = 0} \quad \text{...(1)}$$
$$I_1 I_2 \omega^2 - K_T (I_1 + I_2) = 0$$
$$\omega_2 = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s} \quad \text{...(2)}$$

Put value of $\omega_1 = 0$ in (VII)

$$a_1 K_T - K_T a_2 = 0$$
$$\boxed{\frac{a_1}{a_2} = 1} \quad \text{...(IX)}$$

From (VIII)

$$\frac{a_1}{a_2} = \frac{K_T - I_2\omega^2}{K_T}$$
$$\frac{a_1}{a_2} = I - \frac{I_2\omega^2}{K_T}$$

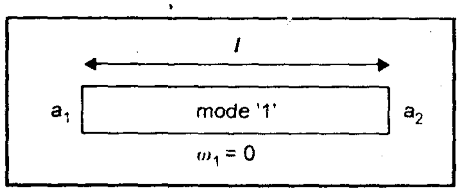
Put value of $\omega_2 = \sqrt{\dfrac{K_T(I_1 + I_2)}{I_1 I_2}}$ rad/s

$$\frac{a_1}{a_2} = 1 - \frac{I_2}{K_T} \times \frac{K_T(I_1 + I_2)}{I_1 I_2}$$

$$\frac{a_1}{a_2} = 1 - \left(1 + \frac{I_2}{I_1}\right) = \frac{-I_2}{I_1}$$

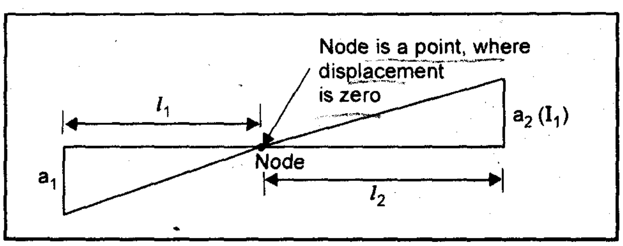
It shows that the angular displacements of rotors are inversely proportional to their inertia.
The section of the shaft where angular displacement is zero Is known as node. First Mode shape

$$\omega_1 = 0, \frac{a_1}{a_2} = 1$$



Second Mode Shape

$$\omega_2 = \sqrt{K_T \frac{(I_1 + I_2)}{I_1 I_2}}, \frac{a_1}{a_2} = \frac{-I_2}{I_1}$$



$$\omega = \sqrt{\frac{K_T(I_1 + I_2)}{I_1 I_2}} \text{ rad/s}$$

2. **Find the frequencies of the system shown in Fig. 5.13.**

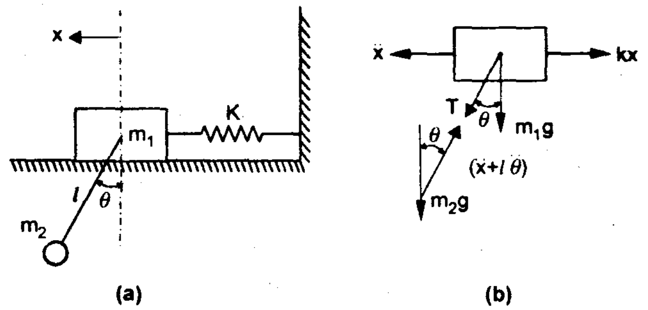


Fig. 5.13

Given : $m_1 = 2$ kg, $m_2 = 0.5$ kg
$K = 90$ N/m, $l = 0.25$ m.

Ans. Initially the pendulum rod is vertical and it is displaced by an angle 0 as shown in figure (a) and free body diagram of forces is shown in figure (b). Let us assume that T is the tension in the pendulum rod.Resolving the forces vertically for m_2

$$m_2 g = T\cos\theta$$

Resolving the forces horizontally, T sin 0 will be known as restoring force as it downwards and brings m_2 to its original state. Horizontal displacement of m_2 is $x + l\sin\theta$.

when θ is very small, $\sin\theta \simeq \theta$ and $\cos\theta = 1$.

So horizontal displacement $= x + l\theta$

and acceleration $= \ddot{x} + l\ddot{\theta}$

Horizontal force $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

So $m_2 g = T$ and $m_2(\ddot{x} + l\ddot{\theta}) = -T\theta$

or $m_2(\ddot{x} + l\ddot{\theta}) + T\theta = 0$

$m_2(\ddot{x} + l\ddot{\theta}) + m_2 g\theta = 0$, put $T = m_2 g$

$(\ddot{x} + l\ddot{\theta}) + g\ddot{\theta} = 0$

or $l\ddot{\theta} + g\theta = -\ddot{x}$

Consider forces for mass m1. All the forces are acting horizontally,

$$m_1 \ddot{x} = -kx + T\sin\theta$$
$$= -kx + T\theta$$
$$m_1 \ddot{x} + kx - T\theta = 0$$

Putting $T = m_2 g$

$$m_1 \ddot{x} + kx - m_2 g\theta = 0$$
$$m_1 \ddot{x} + kx = m_2 g\theta$$

Let us assume the solution of the form

$$x = A \sin \omega t$$

and $\theta = \phi \sin \omega t$

Substituting these solutions in the above two equations, we get

$$-l\omega^2 \phi + g\phi - \omega^2 A = 0$$

and $-m_1 \omega^2 A + kA - m_2 g\phi = 0$

$$\frac{A}{\phi} = \frac{-l\omega^2 + g}{\omega^2} = \frac{m_2 g}{k - m_1\omega^2}$$

The frequency equation can be written as

$$(-l\omega^2 + g)(k - m_1\omega^2) - \omega^2 m_2 g = 0$$
$$-kl\omega^2 + m_1 l\omega^4 + gk - m_1 g\omega^2 - \omega^2 m_2 g = 0$$
$$\omega^4 - \frac{(kl + m_1 g + m_2 g)\omega^2}{m_1 l} + \frac{gk}{m_1 l} = 0$$

So, $$\omega^2 = \frac{(m_1 + m_2)g + kl \pm \sqrt{[(m_1 + m_2)g + kl]^2 - 4m_1 lkg}}{2m_1 l}$$

Substituting the numerical values in the above equation

$$\omega^2 = \frac{(2 + 0.50)\,9.81 + 90 \times 0.25 \pm \sqrt{[(2 + 0.5)9.81 + 90 \times 2.5]^2 - 4 \times 2 \times 0.25 \times 90 \times 9.81}}{2 \times 2 \times 0.25}$$

$$= 24.5 + 22.5 \pm \sqrt{(24.5 + 22.5)^2 - 1764}$$

$$= 47 \pm \sqrt{2209 + 1764} = 47 \pm 21.095$$

$$\omega_1 = 8.25 \text{ rad/sec}, \omega_2 = 5.08 \text{ rad/sec.}$$

3. **Find the natural frequencies of the there is no slip between cord and cylinder system shown in Fig. 5.14. Assume that**

Given : $k_1 = 40$ N/m

$k_2 = 60$ N/m

$m_1 = 2$ kg

$m_2 = 10$ kg

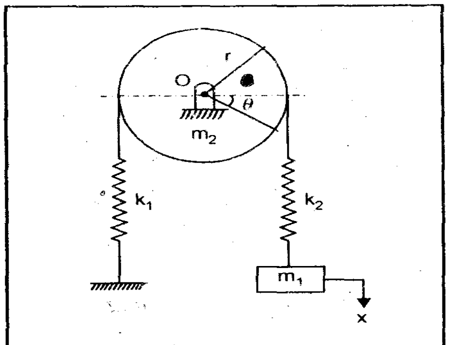


Fig. 5.14

Ans. Let us give x vertical displacement to mass as shown. Since there is no slip between the cord and cylinder, so vertical displacement x causes the cylinder to rotate by angle 0.
Writing the equations

$$m_1 \ddot{x} = -k_2 (x - r\theta)$$

and

$$I\ddot{\theta} = k_2(x - r\theta) r - k_1 r^2\theta$$

where $I = \frac{1}{2} m_2 r^2$ = moment of inertia of cylinder

Above equation becomes

$$m_1 \ddot{x} + k_2 x - k_2 r\theta = 0$$

$$I\ddot{\theta} + (k_1 r^2 + k_2 r^2)\theta - k_2 x r = 0$$

Let us assume the solution of the form

$$x = A \sin \omega t, \ \ddot{x} = -\omega^2 A \sin \omega t$$

$$\theta = \phi \sin \omega t, \ \ddot{\theta} = -\omega^2 \phi \sin \omega t$$

Substituting these values in the above equations

$$-\omega^2 A m_1 + k_2 A - k_2 r\phi = 0$$
$$-\omega^2 I\phi + (k_1 r^2 + k_2 r^2)\phi - k_2 A r = 0$$

$$(k_2 - \omega^2 m_1) A - k_2 r\phi = 0, \quad \frac{A}{\phi} = \frac{k_2 r}{k_2 - \omega^2 m_1}$$

$$(k_1 r^2 + k_2 r^2 - \omega^2 I)\phi - k_2 r A = 0, \quad \frac{A}{\phi} = \frac{k_1 r^2 + k_2 r^2 - \omega^2 I}{k_2 r}$$

$$-k_2^2 r^2 + (k_2 - \omega^2 m_1)(k_1 r^2 + k_2 r^2 - \omega^2 I) = 0$$

Also $I = 1/2\, m_2 r^2$

$$-k_2^2 r^2 + k_1 k_2 r^2 + k_2^2 r^2 - k_2 \frac{1}{2} m_2 r^2 \omega^2 - \omega^2 k_1 m_1 r^2 - \omega^2 m_1 k_2 r^2 + \omega^2 m_1 k\omega^2 \frac{1}{2} m_2 r^2 = 0$$

$$\omega^4 \frac{m_1 m_2 r^2}{2} - \omega^2 \left(\frac{k_2 m_2 r^2}{2} + k_1 m_1 r^2 + m_1 k_2 r^2 \right) + k_1 k_2 r^2 = 0$$

or $$\omega^4 - \omega^2 \left(\frac{k_2 m_2 r^2}{m_1 m_2 r^2} + \frac{2k_1 m_1 r^2}{m_1 m_2 r^2} + \frac{2m_1 k_2 r^2}{m_1 m_2 r^2} \right) + \frac{2k_1 k_2 r^2}{m_1 m_2 r^2} = 0$$

$$\omega^4 - \omega^2 \left(\frac{k_2}{m_1} + \frac{2k_1}{m_2} + \frac{2k_2}{m_2} \right) + \frac{2k_1 k_2}{m_1 m_2} = 0$$

$$\omega^4 - \omega^2 \left[\frac{2(k_1 + k_2)}{m_2} + \frac{k_2}{m_1} \right] + \frac{2k_1 k_2}{m_1 m_2} = 0$$

Substituting the values of various parameters

$$\omega^4 - \omega^2 \left[\frac{2(40+60)}{10} + \frac{60}{2} \right] + \frac{2 \times 40 \times 60}{2 \times 10} = 0$$

$$\omega^4 - \omega^2 (20 + 30) + 240 = 0$$
$$\omega^4 - 50\omega^2 + 240 = 0$$

$$\omega^2 = \frac{50 \pm \sqrt{2500 - 960}}{2} = \frac{50 \pm 39.24}{2}$$

$$\omega_1 = \sqrt{44.62} \text{ rad/sec} = 6.68 \text{ rad/sec}$$

$$\omega_2 = \sqrt{5.38} \text{ rad/sec} = 2.32 \text{ rad/sec}$$

4. Find the natural frequency of the system shown in Fig 5.17.

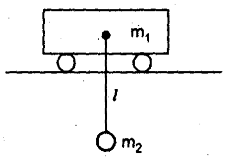


Fig. 5.17

Ans. Let us assume the whole system is moved to the right by x. The ball m_2 is displaced by θ as shown in figure 5.18. The total movement of ball m_2 is $x + l\theta$.

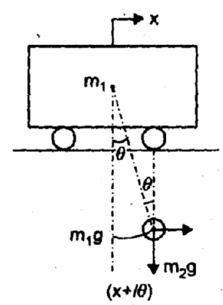


Fig. 5.18

The equations of motion are

For pendulum,

$$m_2(\ddot{x}+l\ddot{\theta}) = -T\theta \qquad (\sin\theta = \theta)$$

$$(T = m_2 g)$$

$$m_2(\ddot{x}+l\ddot{\theta}) + m_2 g\theta = 0$$

$$\ddot{x} + l\ddot{\theta} + g\theta = 0 \qquad ...(1)$$

For mass m_1,

$$m_1\ddot{x} = T\theta$$

$$m_1\ddot{x} - m_2 g\theta = 0$$

or

$$\ddot{x} = \frac{m_2}{m_1}g\theta \qquad ...(2)$$

Putting the value of $\ddot{x}$ from equation (2) in equation (1), we get

$$\frac{m_2}{m_1}g\theta + l\ddot{\theta} + g\theta = 0$$

$$l\ddot{\theta} + \left(g+\frac{m_2 g}{m_1}\right)\theta = 0$$

$$\ddot{\theta}+\left(\frac{g}{l}+\frac{m_2 g}{m_1 l}\right)\theta = 0$$

$$\ddot{\theta}+\frac{g}{m_1 l}(m_1+m_2)\theta = 0$$

So

$$\omega_n = \sqrt{\frac{g}{m_1 l}(m_1+m_2)}$$

5. **Two bodies having equal masses as 60 kg each and radius of gyration 0.3 m are keyed to both ends of a shaft 0.80 m long. The shaft is 0.08 m in diameter for 0.30 m length, 0.10 diameter for 0.20 m length and 0.09 m diameter for rest of the length. Find the frequency of torsional vibrations.**

Take $G = 9 \times 10^{11}$ N/m^2

Ans.

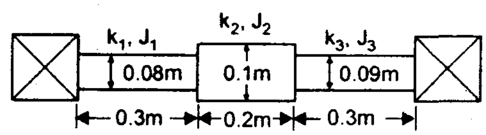


Fig. 5.15

$$I = mk^2 = 60 \times .3 \times .3 = 5.4 \text{ kg m}^2$$

$$k_1 = \frac{GJ_1}{l_1} = \frac{9\times10^{11}\times(\pi/32)\times(0.8)^4}{.30}$$

$$= 1.2057 \times 10^7 \text{ N}-m/\text{rad}$$

$$k_2 = \frac{GJ_2}{l_2} = \frac{9\times10^{11}\times(\pi/32)\times(0.10)^4}{.2}$$

$$= 4.415 \times 10^7 \text{ N-m/rad}$$

$$k_3 = \frac{GJ_3}{l_3} = \frac{9\times10^{11}\times(\pi/32)\times(.09)^4}{.3}$$

$$= 1.93139 \times 10^7 \text{ N-m/rad}$$

where I = mass moment of inertia, J_1, J_2 and J_3 are polar moment of inertia.
The equivalent stiffness of the shaft is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Parts of shaft are connected in series

$$= \left(\frac{1}{1.2057} + \frac{1}{4.415} + \frac{1}{1.93139}\right)10^{-7}$$

$$= (0.829 + .2265 + .51776)10^{-7}$$

$$k = 6.356 \times 10^{6} \text{ N-m/rad}$$

$$\omega = \sqrt{\frac{k(I_1 + I_2)}{I_1 I_2}} = \sqrt{\frac{2k}{I}} \qquad \text{if } I_1 = I_2$$

$$\omega = \left(\frac{2 \times 6.356 \times 10^{6}}{5.4}\right)^{\frac{1}{2}} = 1.534 \times 10^{3} \text{ rad/sec.}$$

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6. Derive the equation of motion of the system shown in figure below and find its frequencies.

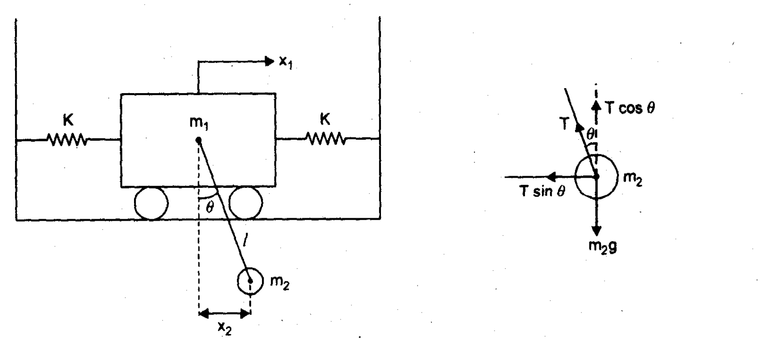

Fig. 5.22

Ans.

$$m_1 \ddot{x}_1 = -2Kx + T\sin\theta$$

$$m_1 \ddot{x}_1 + 2Kx - T\sin\theta = 0$$

$$m_1 \ddot{x}_1 + 2Kx - T\theta = 0 \qquad \text{...(I)}$$

Now $m_2 g = T\cos\theta$

When θ is small, $\cos\theta = 1$

$$m_2 g = T$$

Putting in equation (I), we get

$$m_1 \ddot{x}_1 + 2Kx - m_2 g\theta = 0 \qquad \text{...(II)}$$

$$m_2 \ddot{x}_2 = -\text{T} \sin\theta$$

$$m_2 (\ddot{x}_1 + l\ddot{\theta}) = -\text{T}\theta \qquad [x_2 = x_1 + l\theta]$$

$$m_2 (\ddot{x}_1 + l\ddot{\theta}) + \text{T}\theta = 0$$

$$m_2 (\ddot{x}_1 + l\ddot{\theta}) + m_2 g\theta = 0$$

$$l\ddot{\theta} + g\theta = -\ddot{x}_1 \qquad \text{...(III)}$$

$$\omega^2 = \frac{\dfrac{m_1 g + m_2 g + 2\text{K}l}{m_1 l} \pm \sqrt{\left(\dfrac{m_1 g + m_2 g + 2\text{K}l}{m_1 l}\right)^2 - \dfrac{4 \times 2\,\text{Kg}}{m_1 l}}}{2}$$

$$\omega^2 = \frac{m_1 g + m_2 g + 2\text{K}l \pm \sqrt{(m_1 g + m_2 g + 2\text{K}l)^2 - 8\,\text{Kg}\, m_1 l}}{2m_1 l}$$

$$\omega = \left[\frac{m_1 g + m_2 g + 2\text{K}l \pm \sqrt{(m_1 g + m_2 g + 2\text{K}l)^2 - 8\,\text{Kg}\, m_1 l}}{2m_1 l}\right]^{1/2}$$

Let $x_1 = \text{A} \sin\omega t$; $\ddot{x}_1 = -\text{A}\,\omega^2 \sin\omega t$

$\theta = \phi \sin\omega t$; $\ddot{\theta} = -\phi\,\omega^2 \sin\omega t$

Equation (II) becomes

$$-m_1\omega^2\text{A} + 2\text{KA} - m_2 g\phi = 0$$

$$\text{A}\,(2\text{K} - m_1\omega^2) - (m_2 g)\,\phi = 0$$

$$\frac{\text{A}}{\phi} = \frac{m_2 g}{2\text{K} - m_1\omega^2} \qquad \text{...}(i)$$

Equation (III) becomes

$$-l\,\phi\,\omega^2 \sin\omega t + g\,\phi \sin\omega t = \text{A}\,\omega^2 \sin\omega t$$

$$\phi\,(g - l\omega^2) = \text{A}\omega^2$$

$$\frac{A}{\phi} = \frac{g - l\omega^2}{\omega^2} \qquad \ldots(ii)$$

$$\frac{A}{\phi} = \frac{m_2 g}{2K - m_1\omega^2} = \frac{g - l\omega^2}{\omega^2}$$

The frequency equation can be written as

$$m_2 g\omega^2 = 2Kg - 2Kl\omega^2 - m_1 g\omega^2 + m_1 l\omega^4$$

$$m_1 l\omega^4 - (m_1 + m_2)\, g\omega^2 - 2Kl\omega^2 + 2Kg = 0$$

$$\omega^4 - \frac{[m_1 g + m_2 g + 2Kl]\,\omega^2}{m_1 l} + \frac{2Kg}{m_1 l} = 0$$

---

7. **Calculate the natural frequency of a shaft of diameter 10 cm and length 300 cm carrying two discs of diameters 125 cm and 200 cm respectively at its ends and weighing 480 N and 900 N respectively. Modulus of the rigidity of the shaft may be taken as 2 x 1011 N/m2.**

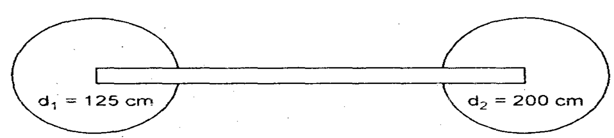

Fig. 5.23

$l = 300 \text{ cm} = 3 \text{ m}$

$d = 10 \text{ cm} = 0.1 \text{ m}$

$w_1 = 480 \text{ N}$

$w_2 = 900 \text{ N}$

$C = 2 \times 10^{11} \text{ N/m}^2$

$$I_1 = \frac{w_1}{g}\frac{r_1^2}{2} = \frac{480}{9.81} \times \left(\frac{62.5}{100}\right)^2 \times \frac{1}{2} = 9.56 \text{ kg.m}^2$$

$$I_2 = \frac{w_2}{g}\frac{r_2^2}{2} = \frac{900}{9.81} \times \left(\frac{100}{100}\right)^2 \times \frac{1}{2} = 45.87 \text{ kg.m}^2$$

$$K_T = \frac{CJ}{l}$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32} (.10)^4 = 9.81 \times 10^{-6} \text{ m}^4$$

$$l = 3 \text{ m}$$

$$K_T = \frac{2 \times 10^{11} \times 9.81 \times 10^{-6}}{3}$$

$$K_T = 6.54 \times 10^5 \text{ Nm/rad}$$

$$w_n = \sqrt{\frac{K_T (I_1 + I_2)}{I_1 I_2}} \text{ rad/s}$$

$$= \sqrt{\frac{6.54 \times 10^5 (9.56 + 45.87)}{9.56 \times 45.87}}$$

$$w_n = 287.52 \text{ rad/s}$$

$$f_n = \frac{w_n}{2\pi} = \frac{287.52}{2\pi} = 45.78 \text{ Hz}$$

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8. What is co-ordinate coupling? Determine the natural frequencies of such system with dynamic coupling?

Ans. Co-ordinate coupling. When we apply brakes on a automobile two motions of car body occur simultaneously.

(1) Translatory (x)

(2) angular.

This type of unbalance occurs on thesystem because centre of gravity (C) of carand centre of rotation do not coincide.

m—Mass of car

I-*MOI

x — Translatory motion

0 — Angular Motion

Equation of motion can be written as :

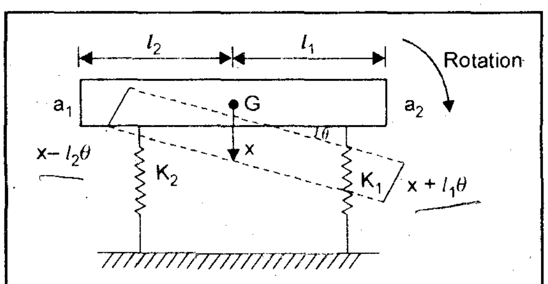

Fig. 5.4

$$m\ddot{x} = -K_2(x - l_2\theta) - K_1(x + l_1\theta)$$

$$m\ddot{x} + (K_1 + K_2)x - (K_2 l_2 - K_1 l_1)\theta = 0$$

$$I\ddot{\theta} = K_2(x - l_2\theta)l_2 - K_1(x + l_1\theta)l_1$$

$$I\ddot{\theta} = (K_2 l_2 - K_1 l_1)x + (K_2 l_2^2 + K_1 l_1^2)\theta = 0 \quad \text{...(II)}$$

= n (I) and (II) are coupled equations as both equations contain x and 0 terms

If

$$K_1 l_1 = K_2 l_2 \text{ then ;}$$

$$m\ddot{x} + (K_1 + K_2)x = 0 \quad \text{...(III)} \quad \text{[complete translatory equation]}$$

$$I\ddot{\theta} + (K_2 l_2^2 + K_1 l_1^2)\theta = 0 \quad \text{...(IV)} \quad \text{[oscillatory equation]}$$

Equation III is of translator nature.

Equation IV is of oscillator nature.

These are uncoupled differential equations and when $K_1 l_1 = K_2 l_2$ then it is called dynamic coupling.

The natural frequencies of the system are:

$$\omega_1 = \sqrt{\frac{K_1 + K_2}{m}} \text{ rad/s}, \quad \omega_2 = \sqrt{\frac{K_1 l_1^2 + K_2 l_2^2}{I}} \text{ rad/s}$$

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9. **What are vibration absorbers ? Prove that spring force of the absorber system is equal and opposite to the excitition force for main system to be stationary?**

Ans. Vibration Absorber. When a structure which is excited by an external harmonic force has undesirable vibrations, it becomes necessary to eliminate them by coupling some vibrating system to it. The vibrating system is known as vibration absorber or dynamic vibration absorber. Vibration absorbers are used to control the structural resonance (consider the main figure)

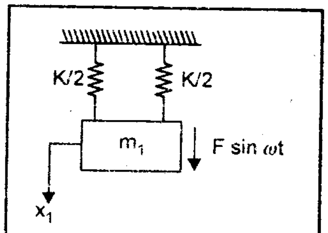

Fig. 5.5 Main System

The natural frequency of this system is $\sqrt{\frac{K}{m_1}}$. When forcing frequency (0 becomes equal to natural frequency of main system then resonance takes place. In order to reduce the amplitude of mass 'nz1' it is coupled with spring mass system (m2 — K2) called Vibration absorber. The spring mass system (ni2 — K2) will acts as vibration absorber andies the amplitude of ni1 to zero if its natural frequency is equal to the excitation frequency

$$\omega = \sqrt{\frac{K_2}{m_2}}.$$

Then, when $\frac{K_1}{m_1} = \frac{K_2}{m_2}$ the absorber is called **tuned absorber.**

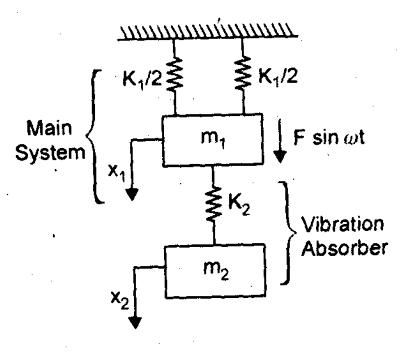

Fig 5.6

Equations of Motion

$$m_1 \ddot{x}_1 = -K_1 x_1 - K_2 (x_1 - x_2) + F \sin \omega t$$

$$m_1 \ddot{x}_1 + (K_1 + K_2)x_1 - K_2 x_2 = F \sin \omega t \quad \text{...(I)}$$

$$m_2 \ddot{x}_2 = -K_2 (x_2 - x_1)$$

$$m_2 \ddot{x}_2 + K_2 (x_2 - x_1) = 0 \quad \text{...(II)}$$

$$x_1 = A_1 \sin \omega t$$
$$x_2 = A_2 \sin \omega t$$

$$m_1 \times -\omega^2 A_1 \sin \omega t + (K_1 + K_2) A_1 \sin \omega t - K_2 A_2 \sin \omega t = F \sin \omega t$$

$$A_1 (K_1 + K_2 - m_1 \omega^2) - K_2 A_2 = F \quad \text{...(III)}$$

$$m_2 \times - A_2 \omega^2 \sin \omega t - + K_2 (A_2 - A_1) \sin \omega t = 0$$

$$- K_2 A_1 + (K_2 - m_2 \omega^2) A_2 = 0 \quad \text{...(IV)}$$

Solving (III) and (IV) for A_1 and A_2. From (IV)

$$A_2 = \frac{K_2 A_1}{K_2 - m_2 \omega^2}$$

Putting in (III)

$$A_1(K_1 + K_2 - m_1\omega^2) - \frac{-K_2^2 A_1}{K_2 - m_2\omega^2} = F$$

$$A_1 [K_1 + K_2 - m_1\omega^2] [K_2 - m_2\omega^2] - K_2^2 A_1 = F (K_2 - m_2 \omega^2)$$

$$A_1 [K_1 K_2 - K_1 m_2\omega^2 + K_2^2 - m_2 \omega^2 K_2 - m_1 K_2 \omega^2 + m_1 m_2 \omega^4 - K_2^2] = F (K_2 - m_2 \omega^2)$$

$$\boxed{A_1 = \frac{F(K_2 - m_2\omega^2)}{\beta}} \quad \text{...(V)}$$

Where

$$\beta = [m_1 m_2 \omega^4 - [m_1 K_2 + m_2 (K_1 + K_2)\omega] + k_1 k_2]$$

$$\boxed{A_2 = \frac{FK_2}{\beta}} \quad \text{...(VI)}$$

In order that amplitude of mass ni1 is zero

Put A1 =0 (so that mass rn1 must not vibrate)

$$K_2 = m_2 \omega^2$$

$$\omega = \sqrt{\frac{K_2}{m_2}} = \omega_2$$

The vibration absorber in which mass and spring constant are selected such that the above condition is satisfied becomes **dynamic vibration absorber.**

Let us assume

$$A_{st} = \frac{F}{K_1} = \text{Static deflection or zero frequency deflection}$$

$$\omega_1 = \sqrt{\frac{K_1}{m_1}} = \text{Natural frequency of main system}$$

$$\omega_2 = \sqrt{\frac{K_2}{m_2}} = \text{Natural frequency of vibration absorber}$$

$$\mu_1 = \frac{m_2}{m_1} = \text{Mass ratio}$$

Multiply N^r and D^r *by* $K_1 K_2$

$$A_1 = \frac{F(K_2 - m_2\omega^2)}{K_1K_2} \Bigg/ \frac{[m_1m_2\omega^4 - [m_1K_2 + m_2(K_1+K_2)]\omega^2] + K_1K_2]}{K_1K_2}$$

$$A_1 = \frac{F/K_1\left(1 - \frac{\omega^2}{\omega_2^2}\right)}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[\frac{m_1}{K_1} + m_2\left(\frac{1}{K_2} + \frac{1}{K_1}\right)\right]\omega^2 + 1}$$

$$\frac{A_1}{A_{st}} = \frac{1 - \frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left(\frac{m_1}{K_1} + \frac{m_2}{K_2} + \frac{m_2}{K_1}\right)\omega^2 + 1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2} - (\omega_1^{-2}+\omega_2^{-2}+\mu\omega_1^{-2})\omega^2+1}$$

$$\frac{A_1}{A_{st}} = \frac{1-\frac{\omega^2}{\omega_2^2}}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VII)}$$

Simlilarly

$$\frac{A_2}{A_{st}} = \frac{K_1K_2}{\beta} \qquad \left[A_2 = \frac{F}{K_1}\ \frac{K_1K_2}{\beta} \text{ where } \frac{F}{K_1} = X_{st}\right]$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^4}{\omega_1^2\omega_2^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+\frac{\omega^2}{\omega_2^2}\right]+1} \qquad \text{...(VIII)}$$

when A1 0, from VII

$$\boxed{\omega = \omega_2}$$

At $A_1 = 0, A_2 = ?$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - \left[(1+\mu)\frac{\omega^2}{\omega_1^2}+1\right]+1}$$

$$\frac{A_2}{A_{st}} = \frac{1}{\frac{\omega^2}{\omega_1^2} - (1+\mu)\frac{\omega^2}{\omega_1^2}} = \frac{1}{-\mu\frac{\omega^2}{\omega_1^2}}$$

$$\frac{A_2}{A_{st}} = \frac{-\omega_1^2}{\mu\omega^2}$$

$$A_2 = A_{st} \times -\frac{\omega_1^2}{\mu\omega^2} = A_{st} \times \frac{-\omega_1^2}{\mu\omega_2^2}$$

$$A_2 = \frac{-F}{K_1} \frac{K_1 m_2}{m_1 \mu K_2} = \frac{-F}{K_2} \frac{\mu}{\mu} \qquad [\because \; \omega = \omega_2]$$

$$\left[\mu = \frac{m_2}{m_1}\right]$$

$$\boxed{F = -A_2 K_2}$$

Hence when the amplitude A1 = 0 i.e. main system becomes stationary the spring force of the absorber is equal and opposite to exciting force. The energy of the main system is absorbed by vibration absorber which is also called auxiliary system.

Amplitude of the auxiliary system is inversely proportional to spring constant ‘K2’. This equation is used for design of absorber.
